**Notes on the dependence of RMST on the time horizon**

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Given S(t) for a random variable T > 0.

Consider the family of random variables T\_h = min ( T, h) derived from T introducing a time horizon h.

RMST(h) := E ( T\_h ) = INT 0\_h (S(t) dt)

var( T\_h ) = INT 0\_h ( t\*S(t) dt) – 2\* (INT 0\_h (S(t) dt))^2

RSDST(h) := sqrt( var ( T\_h ) )

1. RMST(h) is monotone increasing; and strictly so if h < sup( t | S(t) > 0).
2. d/dh {RMST(h)} = S(h).
3. RSDST(h) is monotone increasing; and strictly so if h < sup( t | S(t) > 0).
4. d/dh {var( T\_h )} = 2\* S(h) \* ( h – RMST ( h) ) > 0
5. The variation coefficient: RSDST(h)/ RMST(h) is monotone increasing; and strictly so if h < sup( t | S(t) > 0).
6. d/dh {var( T\_h ) / RMST(h)^2 } =

(2\* S(h) \* ( h – RMST ( h) ) \* RMST(h)^2 – 2\* RMST(h) \* S(h) \*var( T\_h )) / RMST(h)^4 =

2\*S(h)/RMST(h)^3 \* [ h\*RMST(h) – RMST(h) ^2 - INT 0\_h ( t\*S(t) dt) + 2\*RMST(h)^2 ]=

2\*S(h)/RMST(h)^3 \* [ INT 0\_h ( h\*S(t) dt) - INT 0\_h ( t\*S(t) dt) ] > 0

The choice of the time horizon only depends on clinical considerations. Given a single S(t) there is no statistically distinguished time horizon.

TODO: Crossing S(t)s – there may be an optimal H …